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Economic Growth Decomposition. An Empirical Analysis Using Bayesian Frontier Approach

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Abstract

This paper presents an empirical analysis of economic growth in respect of its components, namely input change, technological progress and changes in efficiency. In this work the Bayesian Stochastic Frontier method as well as the output change decomposition procedure, are used in order to evaluate their influence on economic growth. The use of panel data in the study allows for a detailed analysis of economic growth in a given economy and enables the search for general patterns that govern the process. The study is carried using a set of sixteen countries over the period 1995 - 2005.

Keywords: economic growth decomposition, Bayesian frontiers, productivity analysis, models for panel data

JEL Classification: C11, C23, O47, O57.

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1 Introduction

Beginning with Gregory King's *Natural and Political Observations and Conclusions upon the State and Condition of England* (1696), economists have used statistics to research the sources and reasons for the economic growth of nations. The aim of this paper is to make a small contribution to this 300-year long endeavour by exploring how the stochastic frontier approach might elucidate some information on the complexity of economic growth and to reveal some of the regularities that govern the process.
In order to facilitate this, a structural decomposition in a panel data framework is employed in the analysis. This allows us to trace back each country's output growth to its components, namely input growth, technological progress and efficiency change. The Bayesian stochastic frontier framework, developed by Koop, Osiewalski, Steel (1999) is applied to model the output levels and growth rates in a panel of sixteen countries over a period of eleven years. Such an approach has several features, which make it more distinguishable among others. Firstly, it yields small-sample inferences that are valid and which are very important, especially given the scarcity of good quality macroeconomic data. Secondly, it makes it relatively easy to impose regularity conditions in the translog production function as used in this study. Thirdly, not only does it allow us to quantify a given country's economic growth structure, but it also allows us to evaluate precision of the decomposition.
The paper is structured as follows. Section 2 provides a brief introductory to related literature on production frontier analysis of economic growth. Section 3 introduces the Bayesian Frontier Model as well as the method for estimation and decomposition. Section 4 of this work presents the panel dataset compiled for the research, while section 5 contains the empirical study. Section 6 forms analytical conclusions of this research together with a short discussion of the main results in respect to other studies in the field.

2 Related literature

Production efficiency analysis can be traced back to Farrell's (1957) pioneer work on measuring productive efficiency of the US agriculture industry. Today two main approaches can be distinguished (see, e.g., Fried, Lovell, Schmidt (2008) for a comprehensive survey of methods): deterministic, which is largely based on Data Envelopment Analysis (DEA) first used by Charnes, Cooper, Rhodes (1978), and a stochastic, also referred to as econometric (Greene (2008)), which was independently developed by Aigner, Lovell, Schmidt (1977) and Meeusen and Van den Broeck (1977).
Färe, Grosskopf, Norris, Zang (1994) apply DEA to an inter-country panel data study. By combining this estimation technique with output decomposition methodology they are able to trace back changes in productivity to mutually exclusive components, namely technical change and efficiency change.
Kumar and Russell (2002) further advance the use of DEA to trace the origin of pro-

ductivity growth. They note that DEA in some cases "fails to identify the 'true' but unknown frontier especially at low capital labour ratios" (p. 532). Moreover, being a deterministic method, DEA does not account for any measurement error. Feeling that this shortcoming is intolerable in macroeconomic studies they use the bootstrap technique to partially handle this problem.

However, Yamamura and Shin (2007) find fault with Kumar and Russell's analysis. They argue that Kumar and Russell ignore any country and time specific effects and do not acknowledge that their estimators may suffer from an omission bias. Moreover, "if they attempt to reduce omission bias, such unobservable effects cannot be captured by using cross section dataset they constructed" (p. 3).

Nonetheless, the work of Kumar and Russell (2002) remains the exemplar for most studies. Henderson and Russell (2005) enrich the analysis by including a human capital proxy in their study. Caselli and Coleman (2006) refocus the analysis on differences in skilled-unskilled labour productivity, relaxing the assumption of perfect substitutability.

However, for the purpose of their article Caselli and Coleman redefine the generally agreed on concept of productivity and total factor of productivity (TFP). By suggesting that each country has a separate technology, they "define the distance in a particular country's input-output set and the world frontier as a difference in technology"; see Badunenko, Henderson, Zelenyuk (2008), p. 484. In most cases, such as the one in this paper, a particular country's input-output set and the world frontier define technical inefficiency. Hence, the difference appears to be purely in definition.

A more recent study by Badunenko, Henderson, Zelenyuk (2008) re-examines Kumar and Russell's findings. In some ways they confirm Kumar and Russell (2002). In others ways they do not.

In contrast to Kumar and Russell who concluded capital accumulation to be the most significant to output growth, Badunenko, Henderson and Zelenyuk (2008) find that the technological change contribution is almost as important (nearly 80% of the capital accumulation contribution). Thus they argue that that "either capital accumulation or technological change can explain most of the positive shift" (p. 463).

Like Kumar and Russell, they find that strong technological advances are seen at high capital to labour ratios. Additionally they find that such, relatively rich, economies have become generally less efficient over time.

Badunenko, Henderson and Zelenyuk as well as Kumar and Russell have three notable differences in their approach in comparison to this research.

Badunenko, Henderson and Zelenyuk's decomposition methodology, similarly to Kumar and Russell, requires assumption on Returns to Scale (RTS), constant in particular (CRS). Badunenko, Henderson and Zelenyuk also examine results' sensitivity to different RTS assumptions (non-increasing and variable returns to scale). Methodology used in this paper does not require any prior assumption on Returns to Scale, though relatively easy such can be imposed. Instead I use a statistical test to check if the data support such an assumption.

Both studies admit the problem of error measurement in DEA method. They propose (and partially implement) the bootstrap techniques for statistical testing. This is common in many recent DEA studies. However, this issue is not a concern for the methodology presented further in this study. By using a Bayesian approach, with a moderate numerical effort, we can acquire full posterior distribution of any quantity in question and make inference on that basis.
Methodology used in their studies requires no functional form specification of production.
In contrast to Färe, Grosskopf, Norris, Zang (1994), Koop, Osiewalski, Steel (1999) propose a stochastic, parametric approach to growth accounting and productivity analysis. They use Bayesian inference, which they argue has several advantages that allow us to: "i) Obtain exact small sample results in a way that is particularly appropriate for the treatment of this paper's very small data set. ii) Focus on any quantity of interest and derive its full posterior distribution; and in particular, the full posterior distribution of any individual efficiency or any function of the parameters in the data. iii) Easily integrate out any nuisance parameters since each is assigned a probability distribution. Thus, we can take into account parameter uncertainty, a characteristic which is bound to be important since the small sample size will tend to prohibit precise estimation. iv) Easily impose (unlike classical methods) economic regularity conditions on the production function"; Koop, Osiewalski, Steel (1999), p. 457.
Furthermore, a more recent econometric study by Kumbhakar and Wang (2005) also uses a parametric approach to macroeconomic productivity analysis (Stochastic Frontier Analysis). Similar to Koop, Osiewalski, Steel (1999) they use translog production function and estimate, using Maximum Likelihood (ML) method, a Stochastic Frontier (SFA) model also in a panel data framework. However, the ML approach to inference has only asymptotic justifications, hardly relevant in small samples. Thus, in this paper the methodology of Koop, Osiewalski, Steel (1999) is used.

3 Methodology

3.1 The Bayesian model

Let Y_{ti}, K_{ti} and L_{ti} be real output, capital and labour in country i ($i = 1, \ldots, N$) at time t ($t = 1, \ldots, T$) respectively. The model takes the following form:

$$Y_{ti} = f_t\left(K_{ti}, L_{ti}\right) \exp\left(\nu_{ti}\right) \tau_{ti}, \tag{1}$$

where τ_{ti} is efficiency ($0 < \tau_{ti} \leq 1$, where one implies full efficiency) and $\exp\left(\nu_{ti}\right)$ formalises the stochastic nature of the frontier. Furthermore, production frontier is assumed to change over time. Using the translog production frontier, a log linear model based on 1 can be written as:

$$
\begin{aligned}
y_{ti} &= x'_{ti}\beta_t + \nu_{ti} - u_{ti}, \\
x_{ti} &= \left(1, k_{ti}, k_{ti}^2, l_{ti}, l_{ti}^2, k_{ti} \cdot l_{ti}\right)', \\
\beta_t &= \left(\beta_{t1}, \beta_{t2}, \beta_{t3}, \beta_{t4}, \beta_{t5}, \beta_{t6}\right)',
\end{aligned}
\tag{2}
$$

where $u_{ti} = -\ln(\tau_{ti})$, $k_{ti} = \ln(K_{ti})$, $l_{ti} = \ln(L_{ti})$ and $y_{ti} = \ln(Y_{ti})$. In order to ensure that capital and labour elasticities are nonnegative at each data point, the following regularity restrictions need to be imposed for each t and i:

$$
\begin{aligned}
El_{K(ti)} &= \tfrac{\partial y_{ti}}{\partial k_{ti}} = \beta_{t2} + 2\beta_{t3}k_{ti} + \beta_{t6}l_{ti} \geq 0, \\
El_{L(ti)} &= \tfrac{\partial y_{ti}}{\partial l_{ti}} = \beta_{t4} + 2\beta_{t5}l_{ti} + \beta_{t6}k_{ti} \geq 0.
\end{aligned}
\tag{3}
$$

Based on Koop, Osiewalski, Steel (1999) and Koop, Osiewalski, Steel (2000a, b) $\nu_{ti}$is are treated as an independent Normal variables with zero mean and unknown variance σ^2. Moreover, u_{ti}s are independent Exponential variables with mean φ^{-1}; they are also independent from the ν_{ti}s. In order to allow the frontier to evolve over time a linear trend is introduced into each parameter of β_t vector (denoted by "LT" hereafter):

$$\beta_t = \dot{\beta} + t\ddot{\beta} \tag{4}$$

Hence, the LT-translog model takes the following form

$$y = \dot{X}\beta + \nu - u \tag{5}$$

where

$$
y = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix}, \dot{X} = \begin{bmatrix} X_1 & 1X_1 \\ \vdots & \vdots \\ X_t & tX_t \\ \vdots & \vdots \\ X_T & TX_T \end{bmatrix}, \beta = \begin{bmatrix} \dot{\beta} \\ \ddot{\beta} \end{bmatrix}, \nu = \begin{bmatrix} \nu_1 \\ \vdots \\ \nu_T \end{bmatrix}, u = \begin{bmatrix} u_1 \\ \vdots \\ u_T \end{bmatrix},
$$

with

$$
y_t = \begin{bmatrix} y_{t1} \\ \vdots \\ y_{tN} \end{bmatrix}, X_t = \begin{bmatrix} x_{t1} \\ \vdots \\ x_{tN} \end{bmatrix}, \dot{\beta} = \begin{bmatrix} \dot{\beta}_1 \\ \vdots \\ \dot{\beta}_k \end{bmatrix}, \ddot{\beta} = \begin{bmatrix} \ddot{\beta}_1 \\ \vdots \\ \ddot{\beta}_k \end{bmatrix}
$$

and k denotes the number of parameters in the standard two-input translog model ($k = 6$, see Equation 3).

Under such model structure elasticities of capital and labour take the following forms:

$$
\begin{aligned}
El_{K(ti)} &= \dot{\beta}_2 + 2\dot{\beta}_3 k_{ti} + \dot{\beta}_6 l_{ti} + \ddot{\beta}_2 t + 2\ddot{\beta}_3 t k_{ti} + \ddot{\beta}_6 t l_{ti}, \\
El_{L(ti)} &= \dot{\beta}_4 + 2\dot{\beta}_5 l_{ti} + \dot{\beta}_6 k_{ti} + \ddot{\beta}_4 t + 2\ddot{\beta}_5 t l_{ti} + \ddot{\beta}_6 t k_{ti},
\end{aligned}
\tag{6}
$$

whereas the measure of Returns to Scale in this LT-translog model is:

$$\begin{aligned} RTS_{ti} &= El_{K(ti)} + El_{L(ti)} = \dot{\beta}_2 + \dot{\beta}_4 + \left(\dot{\beta}_6 + 2\dot{\beta}_3\right) k_{ti} + \left(\dot{\beta}_6 + 2\dot{\beta}_5\right) l_{ti} + \\ &+ \left(\ddot{\beta}_2 + \ddot{\beta}_4\right) t + \left(\ddot{\beta}_6 + 2\ddot{\beta}_3\right) tk_{ti} + \left(\ddot{\beta}_6 + 2\ddot{\beta}_5\right) tl_{ti} \end{aligned}$$

and Constant Returns to Scale (CRS) for all countries at all time correspond to imposing the six restrictions: $\dot{\beta}_2 + \dot{\beta}_4 = 1$, $\dot{\beta}_3 = \dot{\beta}_5$, $\dot{\beta}_6 = -2\dot{\beta}_5$, $\ddot{\beta}_2 = -\ddot{\beta}_4$, $\ddot{\beta}_3 = \ddot{\beta}_5$, $\ddot{\beta}_6 = -2\ddot{\beta}_5$. Moreover the specified model will reduce to: LT-Cobb-Douglas production specification if $\dot{\beta}_3$, $\ddot{\beta}_3$, $\dot{\beta}_5$, $\ddot{\beta}_5$, $\dot{\beta}_6$ and $\ddot{\beta}_6$ equal zero; Cobb-Douglas production specification with a time trend variable (t) if $\dot{\beta}_3$, $\dot{\beta}_5$, $\dot{\beta}_6$ and $\ddot{\beta}_2 \ldots \ddot{\beta}_6$ equal zero; time-invariant Cobb-Douglas production specification if $\dot{\beta}_3$, $\dot{\beta}_5$, $\dot{\beta}_6$ and $\ddot{\beta}_1 \ldots \ddot{\beta}_6$ equal zero; translog production specification with a time trend variable (t) if $\ddot{\beta}_2 \ldots \ddot{\beta}_6$ equal zero and time-invariant translog production specification if $\ddot{\beta}_1 \ldots \ddot{\beta}_6$ equal zero.
The joint distribution representing the full Bayesian model is:

$$f_N^{TN}\left(y|\dot{X}\beta - u, \sigma^2 I_{TN}\right) p(\beta)p(\sigma^{-2})p(\varphi)\prod_{t=1}^{T}\prod_{i=1}^{N} f_G\left(u_{ti}|1,\varphi\right), \tag{7}$$

where $p(\varphi) = f_G\left(\varphi|1, -\ln r_0\right)$, $p\left(\sigma^{-2}\right) = \sigma^2 \exp\left(-\frac{10^{-6}}{2\sigma^2}\right)$, and r_0 is the prior efficiency median. The literature suggests r_0 should be set as a value from $[0.5, 0.9]$ interval; see Osiewalski (2001) or Marzec and Osiewalski (2008). Usually 0.75 or 0.875 is assumed in empirical studies such as this one; see Koop, Osiewalski, Steel (1999), (2000a, b) or Greene (2008). Though it is not the aim of this research to analyse sensitivity of the model to this prior, I considered values ranging from 0.6 to 0.8. Having found that the estimates are not sensitive to r_0 value I set prior efficiency median to 0.7 throughout the study, which is in the middle of the considered interval. This specific value for r_0 has also been used in other studies that use similar methodology; see Marzec and Osiewalski (2008). The prior on σ^{-2} is close to the "usual" noninformative prior for small to moderate values of σ^{-2}; see Koop, Osiewalski, Steel (1999), (2000a, b).
The economic regularity conditions are imposed through $p(\beta)$; if met, $p(\beta) = 1$, zero otherwise. They are met if average elasticities of capital and labour (see Equation 3) for every country and for every year adhere to economic restrictions (are nonnegative).
The joint $(2k + 2 + NT)$-dimensional posterior density of all unobservables, $p\left(\beta, \sigma^{-2}, \varphi, u|data\right)$, is proportional to (7) and does not correspond to any standard multivariate distribution. In order to summarise main properties of the posterior distribution, this work uses Gibbs sampling. In short, this algorithm amounts to repeated drawing from the full conditional posterior distributions provided by Koop, Osiewalski, Steel (1999). Under certain mild assumptions these drawings converge to realisations from the joint posterior distribution; see Tierney (1994). Given this,

any posterior feature of interest can be calculated. I run 500 000 burn-in cycles and 120 000 final (accepted) draws in order to calculate characteristics of the posterior distribution.

3.2 Output growth decomposition

Considering the world frontier as well as the inputs and inefficiencies of country i in two corresponding periods, t and $t+1$, the increase in the log of a country's output can be expressed by:

$$\frac{1}{2}\left(\beta_{t+1}+\beta_t\right)'\left(x_{t+1,i}-x_{ti}\right)+\frac{1}{2}\left(x_{t+1,i}+x_{ti}\right)'\left(\beta_{t+1}-\beta_t\right)+\left(u_{ti}-u_{t+1,i}\right); \tag{8}$$

see Koop, Osiewalski, Steel (1999). Allowing the annual output change to be broken down as

$$OC_{t+1,i}=IC_{t+1,i}xTC_{t+1,i}xEC_{t+1,i}, \tag{9}$$

where annual input change is given as

$$IC_{t+1,i}=\exp\left(\frac{1}{2}\left(\beta_{t+1}+\beta_t\right)'\left(x_{t+1,i}-x_{ti}\right)\right), \tag{10}$$

annual technical change as

$$TC_{t+1,i}=\exp\left(\frac{1}{2}\left(x_{t+1,i}+x_{ti}\right)'\left(\beta_{t+1}-\beta_t\right)\right), \tag{11}$$

and

$$EC_{t+1,i}=\exp\left(u_{ti}-u_{t+1,i}\right), \tag{12}$$

accounts for annual efficiency change. Moreover, productivity change, which is the joint impact of technical change and efficiency change on production change, is given as

$$PC_{t+1,i}=TC_{t+1,i}xEC_{t+1,i} \tag{13}$$

In order to easily interpret the results the following average annual percentage growth rates are used in the analysis: $AEG_i=100x(AEC_i-1)$, $ATG_i=100x(ATC_i-1)$, $APG_i=100x(APC_i-1)$, $AIG_i=100x(AIC_i-1)$, $AOG_i=100x(AOC_i-1)$; AEC_i, ATC_i, APC_i, AIC_i and AOC_i are the geometric averages of annual changes defined in formulas from (8) to (13).

3.3 Methodology discussion

This study follows the methodology introduced by Koop, Osiewalski, Steel (1999), and can be found similar to a more recent study by Kumbhakar and Wang (2005). There are, however, some notable differences to Kumbhakar and Wang's research.

First, apart from inefficiency terms, Kumbhakar and Wang (2005) introduce country-specific two-sided fixed effects; see Greene (2008). They argue that this allows them to account for heterogeneity across their sample. In this study no additional variables are included to account for such country specific (time-invariant) effects. I assume a common technological frontier in a given year. Considering the nature of this particular sample I believe it is appropriate for a number of reasons. In paricular all of these countries are capitalistic, open market economies; all of these countries are global economies actively participating in international trade organisations like OECD or APEC. Many of them are even more economically integrated as members of the European Union; all of these countries have democratic systems; all of these countries have well developed economies and are capable of annually producing good quality macroeconomic statistics allowing for a reliable international comparisons (e.g., members of OECD); also the purpose of this research is to compare all of the sampled economies to the best practice frontier similarly to Badunenko, Henderson Zelenyuk (2008).

Second, I use raw labour values whereas Kumbhakar and Wang (2005) use human capital estimates. They obtain human capital estimates by taking into account years of schooling. In this way data on years of schooling are used as correction factors for labour to obtain the level of human capital for each country. Although this is a common approach, it is being criticized by scholars of economic growth theory; see Gylfason (1999). Thus, I decide to leave it out of the scope of this study.

Third, Kumbhakar and Wang (2005) model time effects through a time trend variable (t). Although, this is a common approach, change in production function parameters, thus technology, over time may be an important factor explaining influence of world technological progress on a given country's productivity. That is why I follow Koop, Osiewalski, Steel (1999) and allow production parameters to evolve in a linear fashion over time.

Fourth, Kumbhakar and Wang's model assumes a non-negative truncated distribution of inefficiency (normal-half-normal model) while I assume exponential distribution (normal-exponential model). This difference has been reported to have little effect on the results; see Greene (2008).

Finally, Kumbhakar and Wang assume that the final time and country specific inefficiency (u_{ti}) is a product of a stochastic, country-specific non-negative variable (u_i) and a deterministic function of time (G_t) common to all countries ($u_i \sim N^+\left(\mu_i, \sigma^2\right)$, and $G_t = \exp\left[\gamma\left(t - \underline{t}\right)\right]$, giving $u_{it} = G_t u_i$, see Kumbhakar and Wang (2005) for more information). Hence, though introducing extensive frontier heterogeneity across countries, the model significantly constrains efficiency evolution over time to be common to all countries in the sample. In this study I allow for a fully country-specific change in efficiency over time which may be essential to a country-specific inference on technical efficiency change impact on economic growth.

4 Macroeconomic data for growth study

Scarcity of good quality data for economic growth studies is an ongoing concern. Such estimates are usually available only for well developed countries leaving out a big portion of the world. Moreover, though methodologies of main macroeconomic output indicators such as Gross Domestic Product, Net Domestic Product or Gross Value Added are fairly well established, the same cannot be said about input factors. Labour indicators are more and more often adjusted to acquire human capital estimates although there is no theoretically sound and generally agreed on measurement standard to do so. Usually education attainment is used as a proxy to assess human capital level and the quality of education attained is not taken into account. Thus, such measure of human capital may appear to be highly biased and far from its true level; see Gylfason (1999). For this reason, this research does not consider human capital as an input factor, focusing in more detailed on raw labour data. Also, the countries considered in the empirical part can be considered similar as regards the education level of the labour force.

Capital stock measurement techniques are very complex and subjects to ongoing changes; see Schreyer (2003) and (2007). In particular, a major change to the methodology occurred when the System of National Accounts (SNA 1993) was introduced. The new way of constructing National Accounts, based on SNA 1993, changed significantly and proved the capital assessment techniques based on former methodology (SNA 1968) to be inconsistent; see Schreyer (2007). This study uses capital stock estimates based on SNA 1993 and other compatible standards (e.g., European System of Accounts ESA 1995). Nonetheless it should be noted that once a new aggregation methodology is introduced in time it may prove the present capital estimates to be inconsistent as well.

Furthermore the variables in question may be subjects to numerous errors due to aggregation process. This issue can be handled by introducing errors-in-variables (EIV) models into a Stochastic Frontier framework; see Dhawan and Jochumzen (1999). This, however, is beyond the scope of this research. Koop, Osiewalski, Steel (2000b) also address this issue by introducing additional explanatory variables to account for differences in the quality of raw input factors among countries. By doing so, they estimate *Effective-Factor Corrections* which directly augment input factors. Moreover they introduce heterogeneity across the sample by allowing the world frontier to shift between country groups rather than allowing each country to have its own individual effect. Technical progress follows an autoregressive process which, in turn, can facilitate a more endogenous-theory-driven interpretation for technical growth. Koop, Osiewalski, Steel (2000b) find that, though complex and parameter rich, their model is strongly supported by the data. This approach could be used in future research. However, Koop, Osiewalski, Steel (2000b) analysed large and very heterogeneous group of countries, representing the whole world, so their approach may prove unnecessarily general and complicated for our more homogenous data set.

This study is based on a sample of sixteen countries analysed over the period from

Table 1: The list of countries and abbreviations

Label	Description		Code	Country
avGVAgr	real Average annual GVA growth (1995 - 2005)	1	AU	Australia
AOG	Posterior mean of Average Output Growth	2	AT	Austria
D(AOG)	Posterior standard deviation of AOG	3	CZ	Czech Republic
AIG	Posterior mean of Average Input Growth	4	DK	Denmark
D(AIG)	Posterior standard deviation of AIG	5	FI	Finland
APG	Posterior mean of Average Productivity Growth	6	DE	Germany
D(APG)	Posterior standard deviation of APG	7	IT	Italy
ATG	Posterior mean of Average Technical Growth	8	JP	Japan
D(ATG)	Posterior standard deviation of ATG	9	KR	Korea
AEC	Posterior mean of Average Efficiency Change	10	NL	Netherlands
D(AEC)	Posterior standard deviation of AEC	11	PL	Poland
EL(K)	Posterior mean of average Capital Elasticity	12	PT	Portugal
D(EL(K))	Posterior standard deviation of EL(K)	13	SI	Slovenia
EL(L)	Posterior mean of average Labour Elasticity	14	SE	Sweden
D(EL(L))	Posterior standard deviation of EL(L)	15	UK	United Kingdom
RTS	Posterior mean of average Returns to Scale (RTS)	16	US	United States
D(RTS)	Posterior standard deviation of RTS			

1995 to 2005. The measures for production output, physical capital stock and labour were acquired from the EU KLEMS Database (March, 2008 release), downloadable from www.euklems.net. Purchasing Power Parities indices (PPP's) were obtained from Eurostat-OECD statistics. This work uses Gross Value Added (GVA) in current international dollars as the production output indicator, fixed capital stock in current international dollars as a measure of physical capital stock and total hours worked by persons engaged (in millions) as labour.
For a comprehensive overview of the methodology, sources and definitions of the EU KLEMS database see O'Mahony, and Timmer (2009), Timmer, O'Mahony, van Ark (2007), and Van Ark, O'Mahony, Ypma (2007). For an introductory to Purchasing Power Parities see *Purchasing power parities - measurement and uses* Schreyer, Koechlin (2002). Moreover, a more detailed methodology overview on Purchasing Power Parities can be found in *Methodological Manual on Purchasing Power Parities* Eurostat-OECD (2006).

5 The empirical study

5.1 Model performance

In order to evaluate the quality of the chosen model I conduct a number of Lindley-type tests, recently used by Marzec and Osiewalski (2008) in the context of stochastic frontier models. Four different types of model simplifications were considered, two of which represent a different production technology function specification: time-invariant Cobb-Douglas specification, Cobb-Douglas specification with a time trend variable (t), LT-Cobb-Douglas specification, time-invariant translog specification and translog specification with a time trend variable (t).

Let γ denote such a subvector of parameters of the full model that the zero restriction ($\gamma^* = 0$) leads to the model simplification under question. Since (for a large enough number of observations) the marginal posterior distribution of γ is approximately Normal with mean $E(\gamma|y, X)$ and covariance matrix $V(\gamma|y, X)$, the quadratic form $\tau(\gamma; y, X) = [\gamma - E(\gamma|y, X)]' V^{-1}(\gamma|y, X) [\gamma - E(\gamma|y, X)]$ has the posterior close to the chi-square distribution with as many degrees of freedom as there are parameters in γ. Table 2 presents the approximate values of the posterior probability that $\tau(\gamma; y, X)$ exceeds $\tau(\gamma^*; y, X)$ (where $\gamma^* = 0$). The results show that the tested value, labelled γ^*, is far in the tail of the posterior distribution of $\tau(\gamma; y, X)$ for the first three simplifications. The Bayesian counterpart of the chi-square test shows that only the translog model with a time trend variable could be a valid simplification of the full model. Furthermore we can test the Constant Returns to Scale (CRS) restrictions on the model which were discussed earlier in the methodology section. The test results suggest a rejection of the CRS hypothesis.

Table 2: Lindley-type testing of model simplifications

	$\tau(\gamma^*)$	$p(\tau(\gamma) > a \vert \text{data}) = \alpha$		$p(\tau(\gamma) > \tau(\gamma^*) \vert \text{data})$	df
		a for $\alpha = 0.05$	a for $\alpha = 0.01$		
Time-Invariant Cobb-Douglas	393.05	16.92	21.67	0	9
Time-trend Cobb-Douglas	159.37	15.51	20.01	0	8
LT-Cobb-Douglas	157.28	12.59	16.81	0	6
Time-invariant translog	95.32	12.59	16.81	0	6
Time-trend translog	5.89	11.07	15.09	0.31	5
LT-translog with global CRS	18.00	12.59	16.81	0.006	6

Note. $\gamma^* = 0$ is the tested value for γ; LT-Cobb-Douglas model is a Cobb-Douglas production type model with a linear trend in all regression parameters. Tested *Time-trend* models include time trend variable (t) to account for time change whereas *Time-invariant* models take no account for time change; df stands for degrees of freedom, CRS stands for Constant Returns to Scale

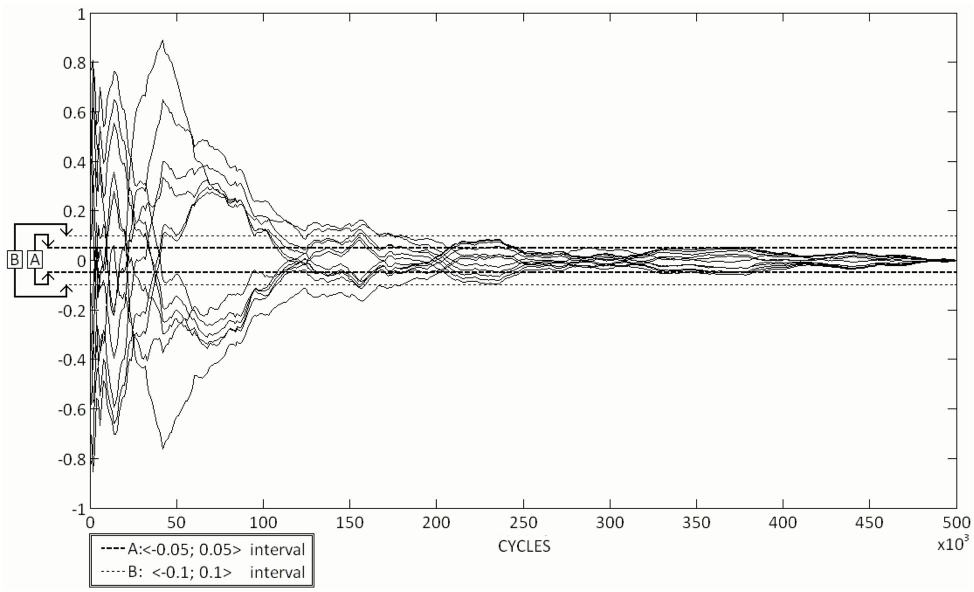
In section 4 I have provided an output growth decomposition framework. The results will be thoroughly analysed in the next part of this paper. Since these estimates are of great importance to the research, a good model should precisely estimate output growth for all countries (very near to the actual GVA growth). In order to assess this more formally, based on Koop, Osiewalski, Steel (2000), I define

$$FIT = 1 - \frac{\sum_{i=1}^{N} \sum_{t=2}^{T} (OG_{it} - \triangle y_{it})^2}{\sum_{i=1}^{N} \sum_{t=2}^{T} (\triangle y_{it} - \triangle \overline{y_i})^2} \tag{14}$$

where OG_{it} is an output growth estimate (posterior mean) in country i in period t given as $OG_{it} = 100x(OC_{it} - 1)$, $\triangle y_{it}$ is the real (observed in the data) GVA growth rate and $\triangle \overline{y_i}$ is the geometric average of GVA growth in country i.

FIT is similar to an R^2. However, since the model has a compound error and one part of it (inefficiency) is included in OG_{it} estimate, FIT value not necessarily has

Figure 1: Monitoring of convergence



Note. CUMSUM statistics for twelve LT-translog parameters; 500 000 burnt-in cycles

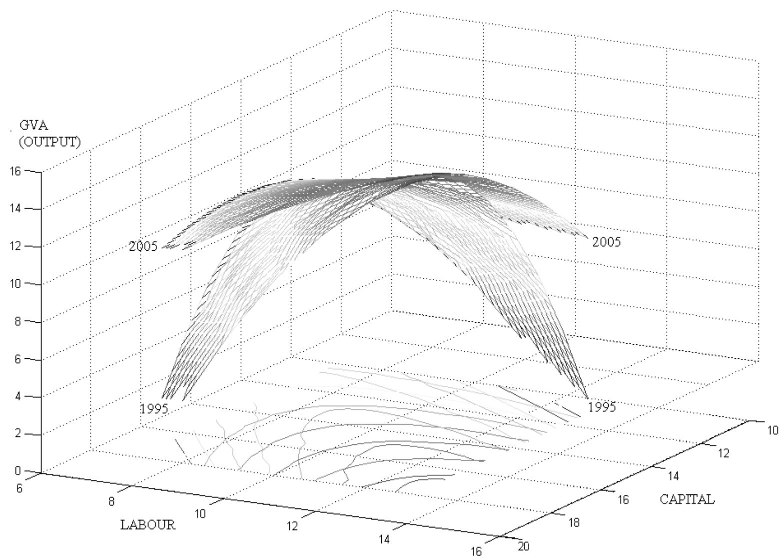
to increase in a more general model; see Koop, Osiewalski, Steel (2000). For the full model used in this study FIT is 0.992.

Finally, application of Markov Chain Monte Carlo methods (MCMC), such as Gibbs sampling, requires monitoring convergence of the chain to its limiting stationary distribution. In order to facilitate this I use CUMSUM statistics proposed by Yu and Mykland (1994), and apply it to the first 500 000 burnt-in cycles. Figure 1 presents a joint plot of CUMSUM statistics for the twelve LT-translog parameters ($\dot{\beta}_1 \dots \dot{\beta}_6$ and $\ddot{\beta}_1 \dots \ddot{\beta}_6$). The graph shows that all series stabilize around zero long before the end of the burn-in process. The $[-0.1; 0.1]$ interval (denoted by B) is not exceeded after approximately 175 000 draws and CUMSUMs permanently stabilize within $[-0.05; 0.05]$ interval (denoted by A) before the algorithm reaches 250 000 burnt-in states. This would provide an empirical evidence of the chain convergence to its limiting stationary distribution.

5.2 Findings

Unless stated otherwise, all point estimates referred to in this section onwards are posterior means. Dispersion measures provided in this paper are posterior standard deviations. For additional information on abbreviations used in the research please

Figure 2: Production frontier surface plots in two corresponding years: 1995 and 2005

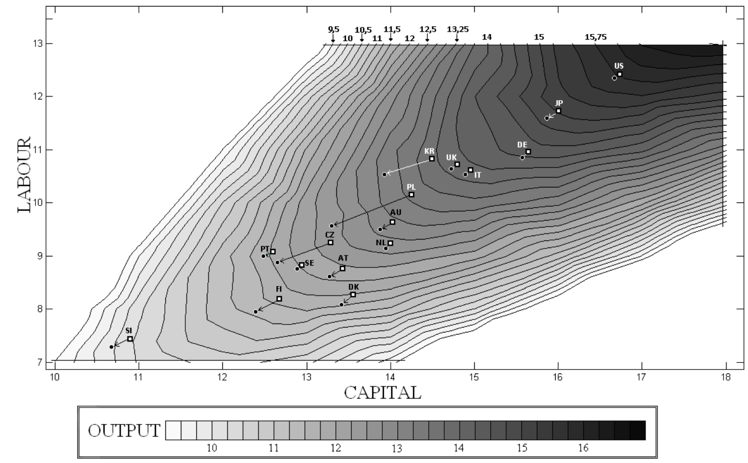


Note. Projection based on model estimates (posterior means); axes in natural logarithms

refer to Table 1. Figures labelled from 2 to 7 present a graphical analysis of the main results summarised in Table 3 and Table 4. It can be noticed that all countries experienced technological progress over time. Technical change, however, had an unequal effect, favouring wealthy economies with high capital-labour ratios (see the last column in Table 4). Figure 2 indicates that the production frontier moderately shifted up and the elasticity of substitution between capital and labour significantly changed over time. This can be seen more explicitly in Figure 3 and Figure 4 which clearly indicate that the elasticity of substitution generally becomes higher over time. Furthermore, the two figures show countries' production capabilities in the two periods, 1995 and 2005, respectively. This, however, should not be mistaken with countries' real production levels at that time, as they were of course lower. The plots allow for an intuitive graphical analysis of such discrepancies between the two conceptual production levels.

The maximum attainable production, given a set of inputs, is indicated by a square whereas a dot shows the measured level. The two figures also provide an overview of the production frontier concept. Namely, the shorter is the arrow pointing from the square to the dot, the closer is the country to its production frontier, or simply the more efficiently it uses its inputs. This also corresponds to efficiency level estimates in Table 4. As we can notice, there are significant differences between the countries' levels of efficiency, which could lead to inconsistent results if the stochastic production

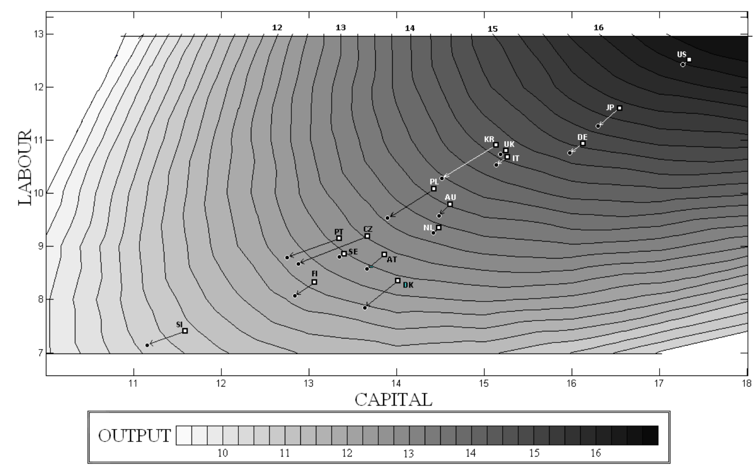
Figure 3: Isoquant map, tear 1995: technical efficiency comparison.



Note. Projection based on model estimates (posterior means); axes in natural logarithms

frontier framework was not introduced in the study.

Figure 4: Isoquant map, year 2005: technical efficiency comparison



Note. Projection based on model estimates (posterior means); axes in natural logarithms

Table 3: Estimation results for output growth decomposition

		DATA	OUTPUT GROWTH DECOMPOSITION									
	Country	avGVAgr	AOG	*D(AOG)*	AIG	*D(AIG)*	APG	*D(APG)*	ATG	*D(ATG)*	AEC	*D(AEC)*
1	AU	5.8893	5.8871	*(0.1219)*	3.3080	*(0.1759)*	2.4968	*(0.2104)*	2.5009	*(0.3261)*	-0.0028	*(0.4202)*
2	AT	4.0642	4.0521	*(0.1198)*	1.7748	*(0.0947)*	2.2377	*(0.1501)*	3.0096	*(0.6035)*	-0.7458	*(0.6170)*
3	CZ	4.5128	4.4900	*(0.1208)*	1.8506	*(0.1998)*	2.5918	*(0.2337)*	1.9212	*(0.2544)*	0.6584	*(0.2442)*
4	DK	4.0087	3.9865	*(0.1211)*	0.8366	*(0.0981)*	3.1239	*(0.1556)*	6.0013	*(3.0638)*	-2.6315	*(2.8729)*
5	FI	5.2161	5.1712	*(0.1202)*	2.2816	*(0.1314)*	2.8253	*(0.1774)*	2.2120	*(0.4849)*	0.6022	*(0.5044)*
6	DE	3.1529	3.1512	*(0.1159)*	0.7323	*(0.4264)*	2.4031	*(0.4497)*	3.7708	*(0.8565)*	-1.3109	*(0.9590)*
7	IT	3.0504	3.0426	*(0.1138)*	1.8557	*(0.1460)*	1.1655	*(0.1766)*	2.2477	*(0.2648)*	-1.0578	*(0.3027)*
8	JP	3.2586	3.2424	*(0.1206)*	0.9268	*(0.6867)*	2.2990	*(0.7041)*	2.7018	*(0.4141)*	-0.3922	*(0.5568)*
9	KR	5.8652	5.7855	*(0.1232)*	4.5337	*(0.4837)*	1.1997	*(0.4818)*	1.8677	*(0.9452)*	-0.6458	*(1.1707)*
10	NL	5.2752	5.2456	*(0.1156)*	1.9581	*(0.1556)*	3.2245	*(0.1932)*	3.4260	*(0.7159)*	-0.1894	*(0.8000)*
11	PL	6.0964	6.0717	*(0.1234)*	0.7391	*(0.1321)*	5.2936	*(0.1850)*	1.8974	*(0.3612)*	3.3341	*(0.3763)*
12	PT	4.6804	4.6723	*(0.1221)*	5.8451	*(0.4989)*	-1.1059	*(0.4794)*	2.0038	*(0.9225)*	-3.0411	*(0.9565)*
13	SI	6.3422	6.3346	*(0.1210)*	5.5677	*(0.8552)*	0.7331	*(0.8311)*	1.7946	*(1.1235)*	-1.0311	*(1.3375)*
14	SE	4.4105	4.4090	*(0.1009)*	2.5295	*(0.2040)*	1.8335	*(0.2231)*	1.9206	*(0.2386)*	-0.0853	*(0.1491)*
15	UK	5.2443	5.2228	*(0.1120)*	2.7456	*(0.2447)*	2.4116	*(0.2688)*	1.9800	*(0.3250)*	0.4242	*(0.4027)*
16	US	5.3915	5.3833	*(0.1131)*	2.8077	*(0.6686)*	2.5096	*(0.6745)*	2.4414	*(0.7813)*	0.0682	*(0.4211)*
	Average	4.7787	4.7592	*(0.1178)*	2.5183	*(0.3251)*	2.2027	*(0.3497)*	2.6061	*(0.7301)*	-0.3779	*(0.7557)*

Note. All point estimates are posterior means; measures of dispersion are posterior standard deviations given in brackets.

The two plots can also be linked with a given country's capital and labour elasticity ratios. Although at first it may seem that most countries were close to having on average Constant Returns to Scale (CRS) the data do not support global CRS model restrictions (see Table 2) and the structure of Returns to Scale (RTS) estimates differs significantly among the studied economies. Those countries that "lay" more towards right-bottom side of the chart present relatively high posterior means of labour elasticity and low capital elasticity (e.g., Denmark and Germany) whereas the ones that are positioned more towards the top-left hand corner represent the opposite situation (e.g., Slovenia and Portugal). Such regularity becomes even more intuitive if analysed from a three-dimension perspective such as in Figure 2. The hump of the stochastic frontier surface creates a fairly straight line. Countries lying within its summit have capital and labour elasticity ratios (El_K/EL_L) close to one (e.g., the United Kingdom). The bigger is the distance from the peak the higher becomes the difference between the two elasticities. The countries with the greatest distance, namely Slovenia and Denmark, are also the ones with the lowest and highest capital to labour ratios respectively (see the last column in Table 4). Interestingly, these are also economies with the lowest posterior means of average RTS in the sample.

5.3 Technical change versus input growth

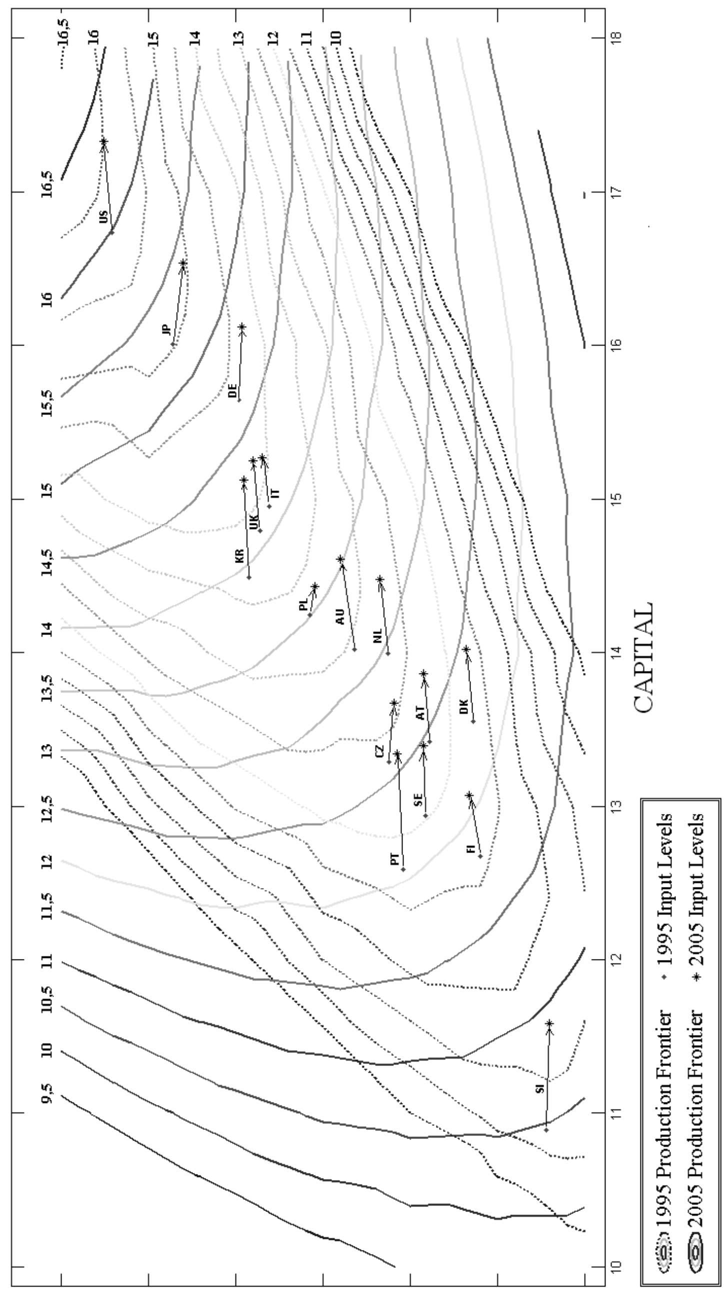
According to the analysis, all countries in 2005 could have had higher production even if their input levels from 1995 where maintained. Hence, technical progress had a common positive influence on economic growth. Furthermore, it is worth mentioning that, though all countries increased their capital stock (accumulated capital), the labour force did not always increase over time. Countries with a decreasing labour force included the Czech Republic, Poland, Germany and Japan; these can be distinguished in Figure 5. The compound graph shows the influence of input change and technical progress on countries' production capabilities over that time. Technical progress is indicated by the isoquants' shape change and shift (moderately up) whereas a country's movement arrow shows input change. This intuitive graph shows a simple relationship. The larger the line movement, the larger was the influence of input growth on a country's production capabilities (e.g., Portugal and South Korea). Furthermore, the more the "input growth path" is parallel to the nearest 1995 isoquants, the more influence had technical progress on a country's economic growth (e.g., Denmark, Austria and the Netherlands).
The pattern is broken when an unusual, high input mix change occurs but does not indicate a straightforward input growth itself. Such a situation occurs when one of the input factors increases over time while the other decreases. Thus, since capital and labour input shifts are not considered separately, the model finds it difficult in such an extraordinary situation to properly distinguish between the influence of technological progress and input change on production output. This results in a high posterior standard deviation of one of the two growth factors estimates. Germany, Japan and Slovenia present such examples.

Table 4: Economic indicators

		EFFICIENCIES			ECONOMIC INDICATORS						CAP.-LABOUR
	Country	AEF	D(AEF)	RANK	EL(K)	D(EL(K)	EL(L)	D(EL(L))	RTS	D(RTS)	RATIOS
1	AU	0.8652	(0.0167)	7	0.3862	(0.0495)	0.5963	(0.0482)	0.9825	(0.0133)	101.5
2	AT	0.8135	(0.0217)	8	0.2984	(0.0658)	0.6476	(0.0766)	0.9460	(0.0311)	127.6
3	CZ	0.4967	(0.0073)	15	0.5456	(0.0581)	0.4262	(0.0598)	0.9718	(0.0175)	72.5
4	DK	0.7172	(0.0648)	13	0.0238	(0.1415)	0.8927	(0.1653)	0.9166	(0.0606)	241.5
5	FI	0.7875	(0.0283)	10	0.4108	(0.0671)	0.5207	(0.0800)	0.9315	(0.0312)	101.4
6	DE	0.9140	(0.0425)	6	0.2133	(0.1072)	0.8070	(0.0974)	1.0203	(0.0201)	143.6
7	IT	0.9453	(0.0152)	4	0.4200	(0.0734)	0.5991	(0.0777)	1.0191	(0.0094)	86.6
8	JP	0.7909	(0.0201)	9	0.3465	(0.1212)	0.7059	(0.1253)	1.0524	(0.0182)	106.1
9	KR	0.5379	(0.0258)	14	0.6468	(0.1113)	0.3894	(0.1311)	1.0362	(0.0283)	53.4
10	NL	0.9408	(0.0275)	5	0.2513	(0.0714)	0.7116	(0.0732)	0.9629	(0.0284)	141.8
11	PL	0.4936	(0.0099)	16	0.5506	(0.0698)	0.4556	(0.0773)	1.0062	(0.0136)	67.9
12	PT	0.7323	(0.0317)	12	0.7446	(0.1040)	0.2325	(0.1053)	0.9770	(0.0284)	49.6
13	SI	0.7490	(0.0559)	11	0.7832	(0.1406)	0.1330	(0.1411)	0.9162	(0.0435)	48.4
14	SE	0.9817	(0.0088)	1	0.5197	(0.0569)	0.4373	(0.0598)	0.9570	(0.0211)	77.6
15	UK	0.9527	(0.0195)	3	0.5080	(0.0846)	0.5186	(0.0961)	1.0266	(0.0151)	72.1
16	US	0.9657	(0.0245)	2	0.3506	(0.1515)	0.7317	(0.1640)	1.0823	(0.0300)	99.6
	Average	0.7927	(0.0263)	-	0.4375	(0.0921)	0.5503	(0.0986)	0.9878	(0.0256)	99.4

Note. Average efficiency level, average elasticities of capital EL(K) and labour EL(L), and average Returns to Scale RTS over the analysed period are posterior means with posterior standard deviation given in brackets. Complementary data on capital-labour ratios are not model estimates; calculated by dividing average capital level by average labour level in 1995-2005.

Figure 5: The compound graph



Note. Isoquant maps and countries production capabilities in two corresponding years: 1995 and 2005; projection based on posterior means; axes in natural logarithms

German and Japanese physical capital stock rose significantly while labour force moderately decreased leading to a considerable change in their input structure composition. The way the two moved their production capabilities along the 1995 isoquants suggests a strong positive impact of technology progress on their output growth. However, the role of their input change remains uncertain. This is indicated by high posterior standard deviations of their average input growth estimates (AIG's, see Table 3). A further decomposition allowing us to separate the influence of capital and labour input on output growth might elucidate some additional information on such phenomenon. I leave it for further study.
Slovenia is another country which significantly increased its capital input with a slight decrease of its labour input. Between 1995 and 2005, however, its input levels moved, rapidly "cutting through" (instead of being parallel) the 1995 isoquants, indicating a strong positive influence of input change on the economic growth. However, the influence of technology, though most likely positive, remains uncertain in this case. This is also indicated by Slovenia's posterior mean of ATG being the smallest in the sample (1.7946), with a high posterior standard deviation (1.1235).
Apart from the three above-mentioned countries, Poland and the Czech Republic have also experienced an abnormal input mix change. However, the two countries' input change was moderate (their "input travel" lines are the shortest in the sample). This allowed the model to properly gauge the input factors' influence on economic growth. This is confirmed by the accuracy of the AIG estimates; posterior mean is 0.7391 ($\pm$0.1321) for Poland and 1.8506 ($\pm$0.1998) for Czech Republic. Moreover, the two countries stand out in their own ways since they both experienced the highest and precisely estimated, increase in production efficiency.
Further analysis of the results shows an additional pattern concerning the influence of technological progress on the elasticity of substitution between the factors of production, namely physical capital stock and labour. As mentioned before technical progress had the biggest influence on those countries in the sample with high capital to labour ratios. This is most likely due to the fact that labour's potential increase, in the long run, is limited by the population size, its structure, social conduct, work culture and other demographic factors which are generally regarded as beyond the reach of economic policy; see Gylfason (1999). In contrast, capital stock can be stimulated by a country's economic system. Hence, through technical progress, which increased the elasticity of substitution between capital and labour, those countries were able to achieve economic growth. The finding becomes even more interesting when we point out those economies for which such technical change was vital to maintain output growth, namely USA, Germany, the Netherlands, Denmark, Austria and Japan. These countries represent the three wealthiest and the most industrialised areas of the world, generally regarded as the main drivers of the world technical progress. Thus, it is of no surprise that the world technical progress stimulates their output growths when they accumulate more capital. This indicates a sound, logical pattern. It seems that in rich, high-developed countries economic growth relies heavily on the introduc-

tion of new physical capital. As technology progresses it allows for a better use of the additional capital accumulated. The new, technologically advanced capital increases the productivity of labour force, which uses it, or even allows for the substitution of labour in the production process. Such new physical capital of course reports much higher capital stock in terms of value than its "used up" predecessors, directing the economy further towards higher capital to labour ratio. All the countries mentioned above seem to undergo such a process. Unfortunately, the role of human capital here remains a mystery. One might assume that since labour uses more and more physical capital the rate of human knowledge in such economies must increase as well. Hence, incorporating human capital into the study should compensate some of the effect of augmenting the elasticity of substitution over time. However, to verify, one would need to introduce a sound measure of human capital which itself is a very complex task; see Gylfason (1999). This is not the aim of this research.

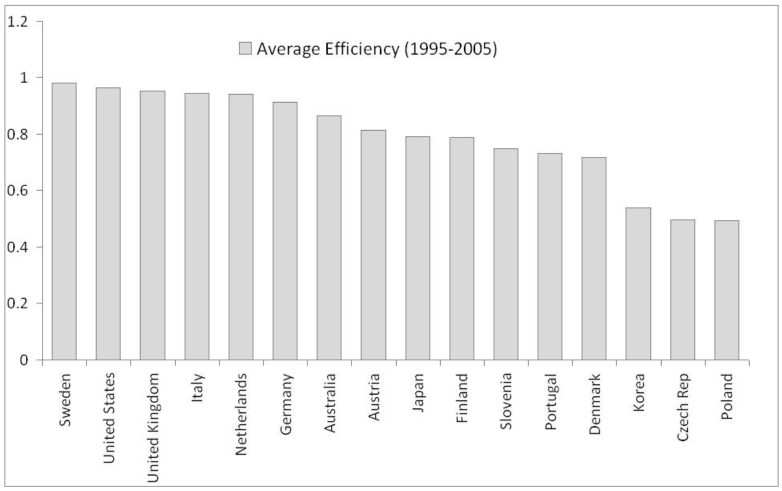
5.4 Efficiency analysis

Figure 6 and Figure 7 present average efficiency levels of countries and their changes between 1995 and 2005. The analysis reveals that the most efficient country in the sample is Sweden. Though at first it may seem a bit odd, the Swedish phenomenon becomes clear when we look at it in more detail. The Swedish economy is modern and highly industrialised. It has a modern distribution system, excellent internal and external communications, and a skilled labour force. Moreover, the Swedish economy is mainly composed of two highly different types of entrepreneurship. The first group are small, hence, efficient and flexible companies which remain small due to unfavourable fiscal policy. They account for nearly 95% of Swedish output. The second group are big successful international enterprises like Tetra Pak, Electrolux and Ikea. Although efficiency level estimates are fairly precise, the same cannot be said about the direction and pace of their change. For example, when considering the efficiency plunge for Denmark over the studied period, one must take into account that the posterior standard deviation is higher than the absolute value of the posterior mean itself. Though most of the studied countries reported a decline in technical efficiency, as Table 3 indicates, there are only two sets of economies for which estimates of average efficiency change are precise relatively to their posterior standard deviations, allowing for a reliable analysis. Those countries are Poland, Czech Republic, Portugal and Italy. Furthermore, though subject to considerable uncertainty, these results are in line with other studies which also report on average technical efficiency decrease over their analysed periods; see Badunenko, Henderson, Zelenyuk (2008).

5.4.1 Poland and Czech Republic

The study indicates that Poland and the Czech Republic were the least efficient countries, but recorded the highest and precisely measured efficiency growth. Though the model does not imply this, it is surely due to them being closely related, post-

Figure 6: Average efficiency levels



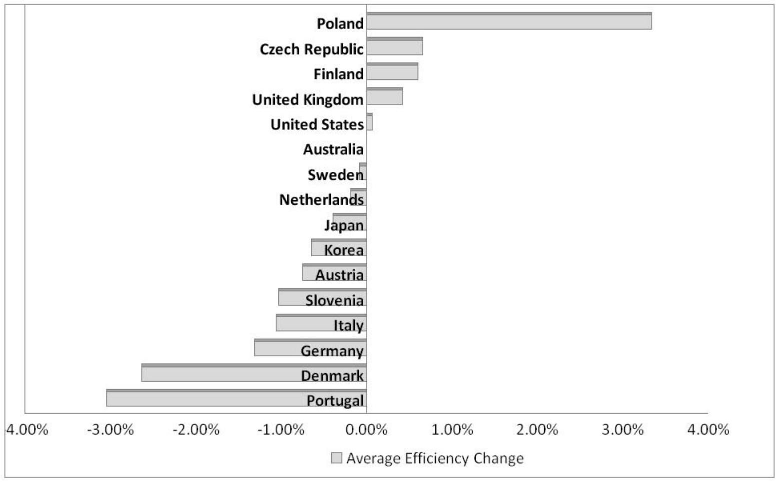
Note. Point estimates based on posterior means. For dispersion measures see Table 4.

communist countries. Centrally planned communist economies were known to be highly inefficient or simply different in terms of their productivity. When setting up production goals there was little interest in an efficient economic outcome. Moreover production was mainly based on big, state-owned inefficient companies with no room for entrepreneurship. Hence, it is of no surprise that pursuing economic liberalisation throughout the 1990s and 2000s, these two countries experienced economic growth, which cannot be explained by input growth or by world technological progress.

5.4.2 Portugal and Italy

Portugal in 2007 was described as a "New sick man of Europe" (The Economist, April 2007). Throughout the studied period (1995 - 2005) it received tremendous financial aid from the European Union to develop its economy by investing in infrastructure, transport, telecommunication and, above all, human capital. The impact of UE policy can be noticed through Portugal's "long input travel line" which is nearly parallel to the "Capital" axis (see Figure 5). This indicates high levels of input growth (mostly capital accumulation driven) which, however, did not go in line with any progress in productivity. Instead Portugal experienced an average decrease in productivity of 1.1059% (±0.4794%) per year. Although being fairly high by world's standards, Portuguese GDP per capita in 2005 was among the lowest in the European Union. In the 2000s, the Czech Republic, Greece, Malta, Slovakia and Slovenia had all overtaken Portugal in terms of GDP per capita and Portuguese GDP per capita had fallen from just over 80% of the EU 25 average in 1999 to just over 70% in 2005. Moreover,

Figure 7: Average efficiency change (AEC)



Note. Point estimates based on posterior means. For dispersion measures see Table 3.

Portugal was the first country to be threatened with sanctions by the European Commission for breaching the euro zone's stability and growth pact, which sets ceilings for euro members' budget deficits. Hence, Portugal is a good example of a country which decreased efficiency contributed to relatively little growth in terms of its potential.
Italy's decreasing economic efficiency could be attributed mainly to what happened after 2001. Around the year 1999 the Italian economy started a decline in productivity and badly needed a dose of pro-market reforms, liberalisation, privatisation, deregulation and a shake-up of the public administration system. All of this was promised in 2001 by Silvio Berlusconi's new government. However, in practice, very little was done to aid the economy (The Economist, November 2005). This might have been the result that from 2001 Italy's productive capabilities were gradually decreasing, even reaching recession in 2003. Nevertheless, the study reports that Italian economy is still very efficient in terms of its level (posterior mean of average efficiency during analysed period is 0.9453; posterior standard deviation is 0.0152).

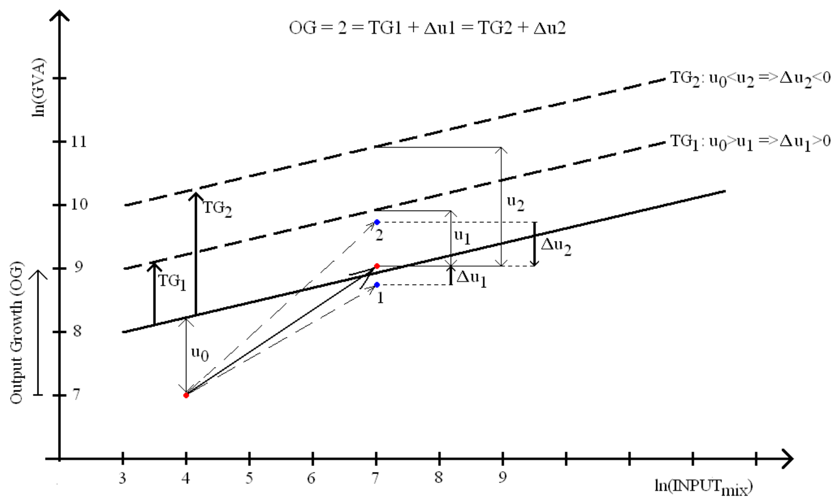
5.5 Distinguishing between technical change and efficiency change

An analysis of Table 3 reveals an interesting pattern. Though the model is very precise in estimating the output growth and decomposing it into input and productivity's contribution to the process, further productivity decomposition is not that accurate. When we look at the low precisions of average efficiency change estimates we can

Table 5: Posterior correlation between average technical growth (ATG) and average efficiency change (AEC)

Country	Posterior correlation
Germany	-0.8273
Denmark	-0.9983
Netherlands	-0.9788

Figure 8: Growth decomposition geometric plot



Note. Case "1": high technical growth induces negative efficiency change; case "2": low technical growth induces positive efficiency change

see just how subtle the technical-efficiency breakdown is. Especially decomposition results for Denmark, the Netherlands and Germany bring attention to the matter. The posterior correlation analysis of ATG and AEC reveals a strong negative correlation between the two variables.

Figure 8 shows a two-dimensional plot, which is a simplified geometric representation of the decomposition structure. When viewing the graph it should be noted that the output growth itself is an observable process (through GVA increase) and the role of the model here is to assess the impact of input, technical and efficiency change on it. Moreover, the input change is also observable, and all countries in a given year are assumed to operate accordingly to the same technology, which is assumed to progress in a linear fashion over time. So, given the world technological progress, the observed input location and growth "path" determines the influence of technical progress on the country's economic growth. This, in turn, is highly dependent on the structure of the model. Hence, in order to arrive at the proper estimate of output growth the plausible high or low estimate of technical growth is compensated by the last ingredient of the decomposition that is the least bounded one by the structure, namely efficiency change. In simple words, if one is set very high the other must be accordingly low to balance the equation to estimated output growth (which is close to observed output growth). Whether this is an artefact of the model or a real relationship between the two variables remains a question. Nevertheless there are some facts in favour of the latter. For example, on the one hand, if a new technology is introduced, switching towards it will definitely produce some efficiency plunge since it takes time to properly use its potential. On the other hand, if technological progress is absent in the economy, it gives the time and opportunity to concentrate efforts on efficiently utilising the available technology. Furthermore, this finding is in line with the general purpose technology argument: it takes time before newly implemented technology can be fully efficiently utilised; see Helpman and Rangel (1999).

5.6 Economic growth: close up on Poland

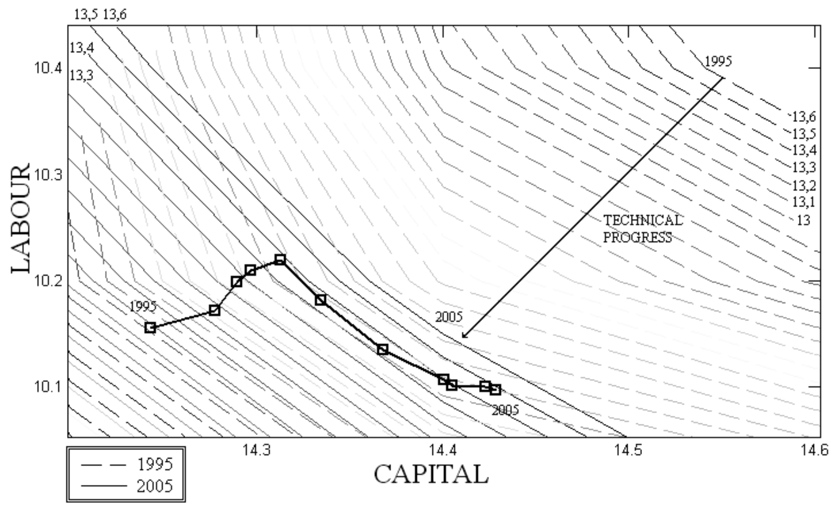
In its essence a frontier analysis involves determining a production frontier based on sampled countries and then measuring distance from the frontier for each country. Although it may be a bit risky to give an interpretation to the model estimates, one may want to reach beyond the model and try to account for the results of output decomposition as reflecting causality.

Since 1989, Poland has been transforming from a centrally-planned economy towards a more efficient liberal one. The study indicates that between 1995 and 2005 Poland has experienced tremendous progress in efficiency, no doubt due to the change in its economic system (see Figure 7).

Throughout the 1990s new liberal Polish laws boosted entrepreneurship and ongoing privatisation made state-owned companies become more efficient. It seems that those factors were crucial to overall high Polish economic efficiency gains since the business sector is now the main growth driver of the Polish economy. However, the pace slowed after 1998.

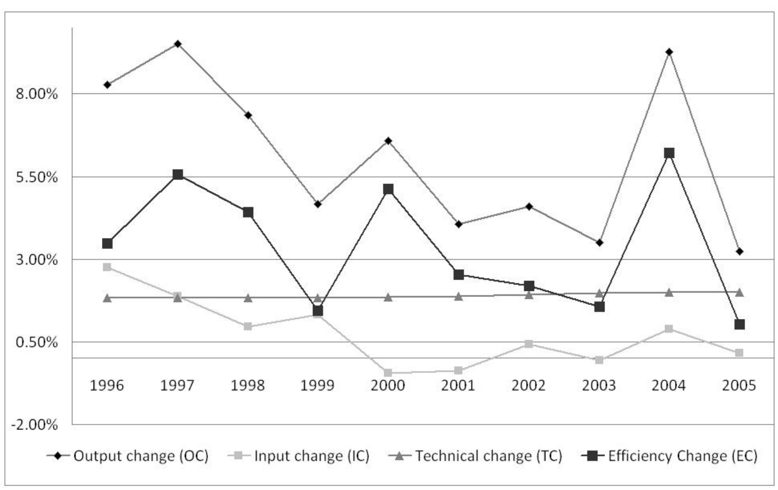
Figure 9 points out that this was the result of an unemployment spike. Unemployment's influence on Polish economic growth can also be seen through a direct input factor decrease on Polish production output in 2000 and 2001 (see Figure 10). Figure 10 also points out the reliance of Polish economic growth on efficiency growth. It seems that efficiency growth was the dominant component in Poland's output growth. Even though labour was on the decline throughout most of the studied period and influence of world technical progress was moderate, Poland still managed to achieve one of the highest posterior means of average growth rate in the sample. As mentioned earlier, Poland experienced a moderate average annual technical progress of 1.897% (ATG posterior mean of 1.8974% with a standard deviation of 0.361%). The model also indicates very moderate (if any) economies of scale of 1.006 (± 0.0136) during the analysed period. This can be investigated thoroughly using Highest Posterior Density (HPD) intervals which facilitate a more detail analysis.

Figure 9: Production frontier growth path; Poland 1995 - 2005



Note. Isoquant maps for 1995 and 2005; dashed lines represent 1995 isoqunt map, solid lines represent 2005 isoquant map; projection based on model estimates (posterior means).

Figure 10: Output growth structure over time; Poland 1995 - 2005



Note. Point estimates based on posterior means

The shortest HPD interval (based on the histogram of the posterior distribution of average RTSPL in the analysed period) containing constant returns to scale (CRS) value is $[0.9997, 1.017]$ and has 53% of the posterior probability. Further details of HPD interval analysis are provided in Figure 11. The results show that (using Lindley-type tests) there is no reason to reject the CRS hypothesis for Poland.

5.7 Economic growth: the United States of America

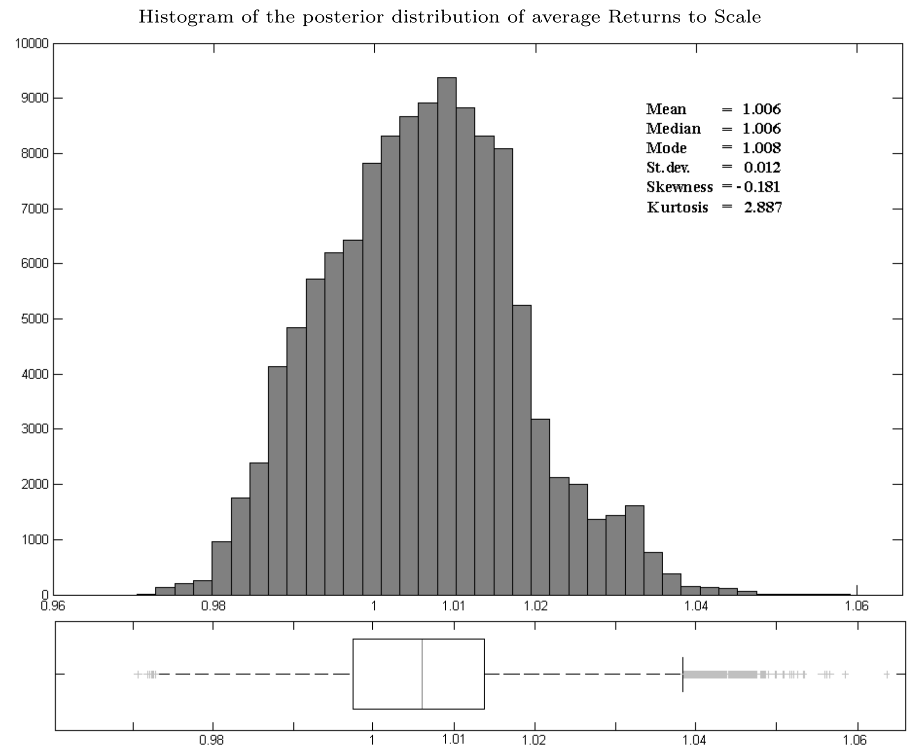
The United States of America is the largest economy in the world and the main driver of world technical progress. According to the research, the U.S. is one of the most efficient countries in the sample and has the highest posterior mean of average RTS of 1.082 (± 0.03).

Unlike most countries in the study, the U.S. did report a slight increase in efficiency. Since the posterior standard deviation of average efficiency change is over six times higher than the mean itself, it indicates that the accuracy of such point estimate leaves much to be desired. Nevertheless even though we are uncertain of the direction (if any) in the change of technical efficiency over time in the U.S., we can be quite certain of its average level over the analysed period. The posterior mean of average efficiency level is nearly forty (39.4) times higher than the posterior standard deviation indicating an accurate result which places the United States of America as the second most efficient economy in the study (see Table 4). Further analysis of the results shows that the U.S. posterior distribution of average $\mathrm{RTS}_{\mathrm{USA}}$ has the highest posterior mean in the sample over the analysed period. Moreover when looking at the histogram of $\mathrm{RTS}_{\mathrm{USA}}$ posterior distribution we can notice that virtually all of the probability mass lies above one. A detailed analysis of average $\mathrm{RTS}_{\mathrm{USA}}$ using Highest Posterior Density intervals can be viewed in Figure 12.
Finally apart from that, Figure 13 and Figure 14 may provide some more insight into how, considering the results, American economy could have maintained its economic growth and what could have been the role of technical progress. By looking at Figure 13 one can notice that the input growth of American economy was mainly the result of capital accumulation (production frontier growth path nearly parallel to the "Capital" axis). One way to account for this would be to argue that since the world technical progress induced an overall increase in elasticity of substitution between capital and labour, it allowed the U.S. economy for a more productive use of the newly accumulated capital.

5.8 Economic growth: South Korea

The study indicates that South Korean posterior mean of average output growth rate was one of the highest even though the country was among the least efficient ones in the study. In order to investigate this phenomenon, once again one needs to reach beyond the model and examine economic factors that would account for such results.

Figure 11: Average returns to scale analysis; Poland 1995 - 2005



Highest Posterior Density intervals

Definition	HPD interval	Posterior probability	Formal definition
The shortest HPD interval containing CRS	$[0.9997, 1.017]$	53%	$P\left(0.9997 \leq RTS_{PL} \leq 1.017\right) = 0.53$
HPD interval containing 90% of posterior probability	$[0.9884, 1.0308]$	90%	$P\left(0.9884 \leq RTS_{PL} \leq 1.0308\right) = 0.9$
HPD interval containing 95% of posterior probability	$[0.9848, 1.0323]$	95%	$P\left(0.9848 \leq RTS_{PL} \leq 1.0323\right) = 0.95$

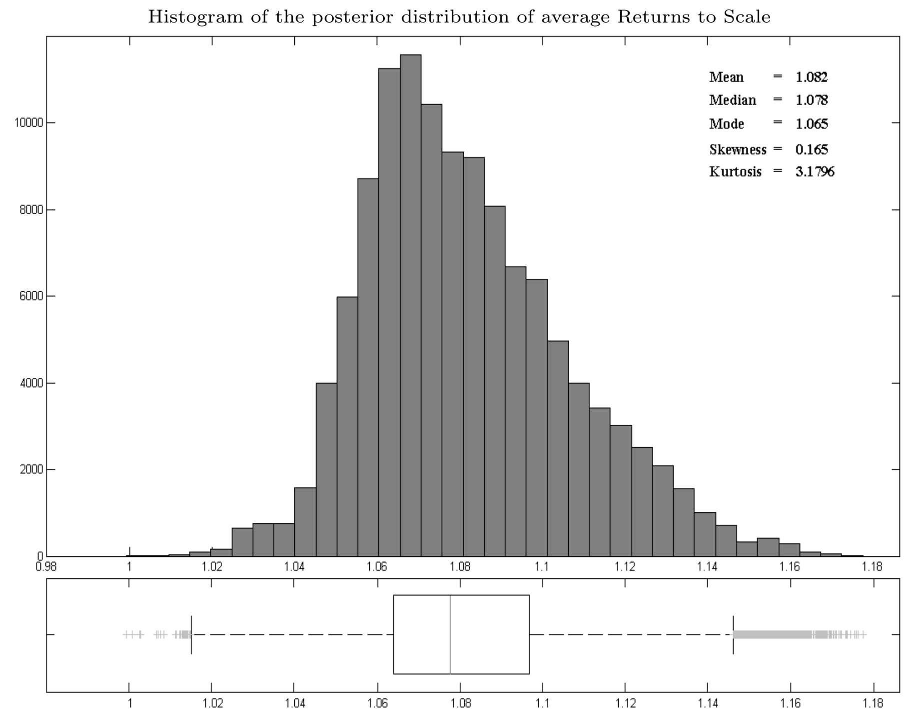
Lindley-type test for average Constant Returns to Scale in Poland

$\tau\left(\gamma^*\right)$	$p\left(\tau\left(\gamma\right) > a \vert \text{data}\right) = \alpha$		$p\left(\tau\left(\gamma\right) > \tau\left(\gamma^*\right) \vert \text{data}\right)$	df
	a for $\alpha = 0.05$	a for $\alpha = 0.01$		
0.19	3.81	6.63	0.662917	1

Note. $\gamma^* = 1$ is the tested value for γ (CRS). Since γ is a scalar, in this case of Lindley-type test the square root of $\tau\left(\gamma; y, X\right)$ has the posterior close to Normal distribution, and thus a simple "t" test could be used instead. Posterior skewness and kurtosis are calculated as third and fourth standardised moments respectively.

Let us consider the way in which the Republic of South Korea achieved such rapid economic success, prior to the IMF crisis. In order to reach high production outputs new capital assets were either acquired through joint ventures or simply through loans. This was funded by state-owned banks, which lent money to business on very

Figure 12: Average returns to scale analysis; USA 1995 - 2005



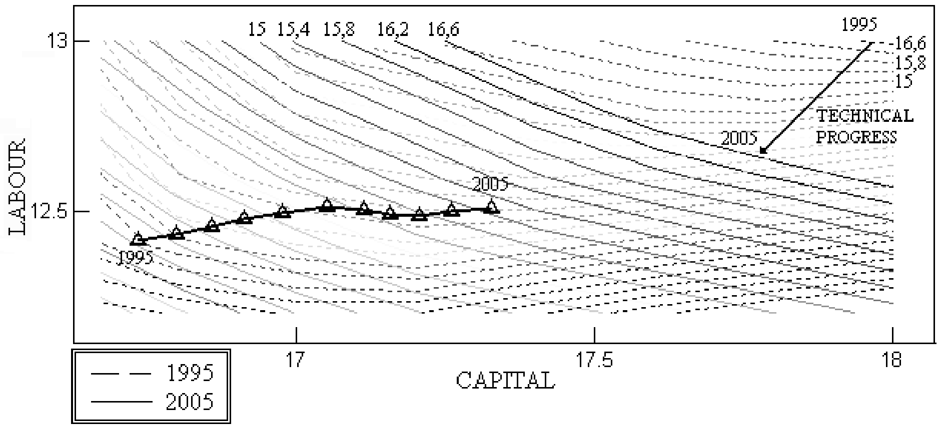
Highest Posterior Density intervals			
Definition	HPD interval	Posterior probability	Formal definition
HPD interval containing 90% of posterior probability	$[1.0482, 1.1278]$	90%	$P(1.0482 \leq RTS_{USA} \leq 1.1278) = 0.9$
HPD interval containing 95% of posterior probability	$[1.0423, 1.1364]$	95%	$P(1.0423 \leq RTS_{USA} \leq 1.1364) = 0.95$

Lindley-type test for average Constant Returns to Scale in USA				
$\tau(\gamma^*)$	$p(\tau(\gamma) > a \mid \text{data}) = \alpha$		$p(\tau(\gamma) > \tau(\gamma^*) \mid \text{data})$	df
	a for $\alpha = 0.05$	a for $\alpha = 0.01$		
7.53	3.81	6.63	0.006068	1

Note. $\gamma^* = 1$ is the tested value for γ (CRS). Since γ is a scalar, in this case of Lindley-type test the square root of $\tau(\gamma; y, X)$ has the posterior close to Normal distribution, and thus a simple "t" test could be used instead. Posterior skewness and kurtosis are calculated as third and fourth standardised moments respectively.

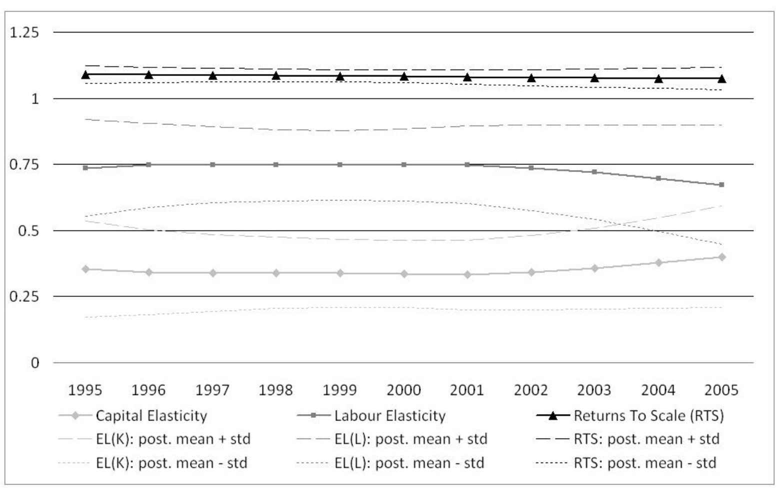
low or even negative real interest rates. Furthermore, labour force costs were kept artificially low. This was achieved through complex and inefficient social conduct. On the one hand, it was virtually impossible in South Korea to lay off an employee. On the other hand it was impossible for an employee to get a raise and the work week was much longer than in most countries in this study; see El-Kahal (2001). Even though these actions provided the desired output growth results, they proved to be disruptive

Figure 13: Production frontier growth path; USA 1995 - 2005



Note. Isoquant maps for 1995 and 2005; dashed lines represent 1995 isoquant map, solid lines represent 2005 isoquant map; projection based on posterior means.

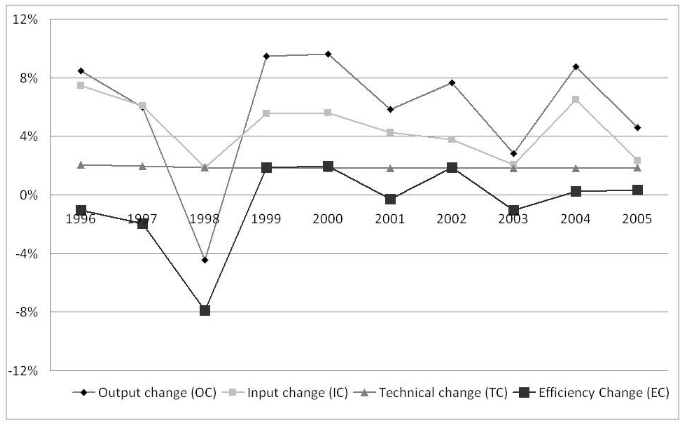
Figure 14: Returns to scale, elasticities of capital and labour; USA 1995 - 2005



Note. Point estimates based on posterior means; std stands for posterior standard deviation.

in the long run, encouraging big conglomerates (known as *chaebols*) to accumulate debt and discouraging them from improving their financial performances. This in turn made South Korean market structure particularly susceptible to international financial turmoil and brought economic ruin during the Asian financial crisis.

Figure 15: Output growth structure over time; South Korea 1995 - 2005



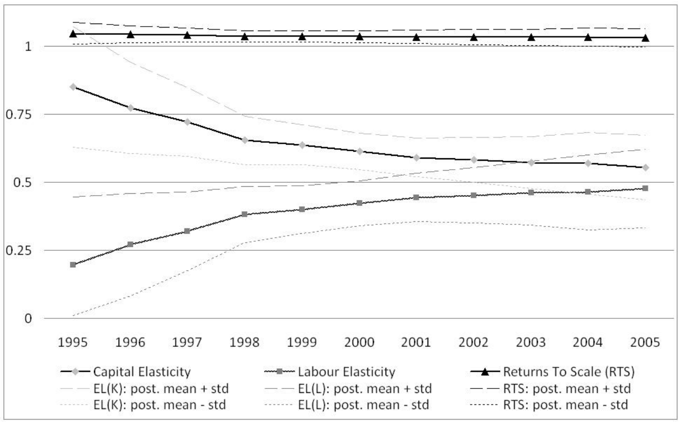
Note. Point estimates based on posterior means.

Looking at the country's growth decomposition figure, one can notice just how severe was the crisis, bringing the production output estimate down from over 8% (±1.28%) annual growth rate in 1996 to a level of over 4% (±1.11%) recession in 1998 (see Figure 15). No doubt that the financial crisis that swept through South Korea in those years is responsible for the biggest production efficiency drop in the panel study. Korean companies were faced with declining demand and thus declining production while being unable to adjust appropriately their inputs. This would account for the fact that in 1998 technical efficiency of South Korean economy plunged by 7.88% (±2.32%).
Thankfully with the help of International Monetary Fund South Korea got out of the crisis relatively fast and continued its impressive growth performance; see The Economist (November 2000). Furthermore, it is worth mentioning that due to the crisis several changes were introduced. The most important was that South Korea significantly liberalised the labour market and has been actively implementing the policy of "The Economy Based on Knowledge" which could have been the reason for a gradual increase in the labour force productivity at the same time diminishing the role of capital accumulation in the macroeconomic production process (see, .e.g., Figure 16).

6 Conclusions

The aim of this study was to elucidate some information on the complexity of economic growth in respect to its components, namely input growth, world technical progress and efficiency change. The study allows us to draw several conclusions on this matter.

Figure 16: Returns to scale, elasticity of capital and labour; South Korea 1995 - 2005



Note. Point estimates based on posterior means; std stands for posterior standard deviation.

First of all, there is no "golden rule" for economic growth. As the study indicates, an increase in each of the factors of the production process has a different impact on economic growth depending on the given economic situation. For example, whether world technological progress matters more than input or efficiency increase on economic growth is solely dependent on a given country's current economic situation and level of development. What can be noted, however, is that in most cases analysed, input growth or technical progress are the main driving forces for output growth whereas efficiency change tends to play a minor, and hard to asses accurately, role (in many cases being more likely to contribute to output decline over time rather than its increase).

Moreover, world technological progress has had a positive influence on all studied economies. However, the biggest beneficiaries of this process are the wealthy, high-developed countries, generally regarded as the world technological leaders. It is of no surprise that technical progress favours these economies and allows them to grow more. Considering this, it should be noted that these countries become more and more dependent on increases in capital (capital accumulation) and the accompanying technological progress. Hence, any ruptures in the process might result in their economic stagnation or probably even recession. This becomes even more disturbing when we consider that the economies discussed here like the USA, Germany, and Japan amount to the world's economic foundations and their problems would quickly transfer to other, dependent economies.

Furthermore, when a significant input change takes place which does not straightforwardly imply a joint capital and labour input growth, it is difficult to assess the impact of input and world technical progress on the given country's economic performance. Such an unusual situation occurred in the case of Germany, Japan and

Slovenia and led to a high posterior standard deviation in one of the two estimates. Nevertheless, the two-way distinction between input growth and productivity growth is relatively precise and generally applicable.

Further productivity decomposition into efficiency and technical change influence, however, is much more subtle and prone to high standard deviations of efficiency change estimates in particular. No doubt, this is due to reported strong negative posterior correlation between the two unobserved variables. The study indicates that the precise efficiency change estimates can be obtained only for those countries which economic performance really stands out, and neither can it be contributed to the world technical progress nor to the change in their input factors.

Nevertheless, efficiency change is an important element, often accounting for an extraordinary economic performance, both in terms of its high output growth rate or lack of it. Thus efficiency change should not be left out in macroeconomic productivity studies such as this one. Especially for countries like Poland and Portugal, it plays an important explanatory role, positive for Poland and negative for Portugal. Therefore in order to reach proper conclusions it is important not only to obtain efficiency growth or decline estimates but also to assess their accuracy (through dispersion measures). Thankfully, by applying Bayesian inference in the research, it is easy to accomplish.

There are significant differences in efficiency levels among analysed countries. The least efficient ones are post-communistic countries (Poland and the Czech Republic) and South Korea, which underwent a huge economic crisis during the analysed period. The most efficient ones are relatively rich and industrialised world economies.

Finally, I have to highlight the fact that studies such as this one will always lack in definite accuracy and confidence of the results. Firstly, due to the fact that macroeconomic datasets are subjects to approximations, errors and omissions. Secondly, because of the methodology used to produce them. The first issue was partly taken into account by introducing stochastic frontier framework in a Bayesian approach. This allowed for assessing the accuracy of estimates and making inference on that basis. The second issue however is much more complex since there are ongoing changes in calculation standards, especially for capital stock.

Interestingly my results place themselves between the two studies of Kumar and Russell (2002) and Badunenko, Henderson, Zelenyuk (2008). In contrast to Kumar and Russell who find that capital accumulation is the principal driving force in the mean growth of worldwide productivity, Badunenko, Henderson and Zelenyuk conclude that technical change is nearly as important. I find that technical change is mostly concerned with the change in isoquants' shape (more than "shift up") over time allowing for a better substitution between capital and labour. Thus, how technological progress influences a given economy depends on (1) its current capital-labour ratio, (2) the rate with which the new input is being accumulated and (3) change in its input structure and thus change in capital-labour ratio over time (whether input growth is labour or capital driven, or both). Therefore to some extend, I concur with Badunenko, Henderson and Zelenyuk's findings since the decomposition results show that either input growth or world technical progress was the main driving force of output growth to many economies in this study. Moreover, considering countries' input growth paths, capital accumulation seems to play key role in input increase. However this should not be treated as a general rule, since how these changes contribute to output growth is dependent on a given country's current economic status. As I presented in the paper, influences of these two factors are dependent on country's placement and "movement" on the (evolving over time) isoquant map which determines the impact the two have on output growth.

Badunenko, Henderson and Zelenyuk also find that "rich countries benefited more from technological change than the poor ones" (p. 463). Again, I find corresponding results since, though all countries in my study benefit from the world technical progress, highly capitalised, rich countries are those for which technical progress is essential to maintain economic growth.

Though the results for technical and efficiency change are in line with Badunenko, Henderson and Zelenyuk as well as Kumar and Russell's research conclusions they should be treated with caution. I find that many point estimates of efficiency change seem to be statistically insignificant (high posterior standard deviations in respect to their posterior means) so it would be unwise to draw firm conclusions only on that basis. Further analysis, however, revealed a strong negative posterior correlation between technical and efficiency change in the wealthy economies strongly influenced by the world technical progress. This would prove Badunenko, Henderson and Zelenyuk's notion about a trade off between high technological progress and technical efficiency.

Moreover Badunenko, Henderson and Zelenyuk argue that poorer countries (with relatively little capital stock to labour) benefit much more from capital accumulation than the rich ones and that most efficiency improvements are reported among poorer countries. I find corresponding results for a number of countries in my study (input driven economic growth of Slovenia and Portugal or efficiency driven growth of Poland and Czech Republic). In contrast to my findings, Kumbhakar and Wang (2005) report a world technical regress. This does not prove wrong any of the two approaches to modelling technical change. As I showed with a Lindley-type test, using a time trend variable to model impact of technical change over time could be a statistically valid simplification. That is why the difference in the estimation results for technical progress should be contributed more to the fact that both models were estimated using different datasets with different timeframes, rather than to the modelling procedure itself. Nevertheless, I find that changing elasticity of substitution (expressed by isoquants' shape change over time) plays an important role in explaining technical progress impact on countries' economic performances.

To conclude this short discussion, like Koop, Osiewalski, Steel (1999), I also must emphasise that, though input growth and technical growth tend to be dominant factors of output growth, all three components play an important role in explaining output change, and thus there seems to be no general pattern that could currently become the basis of universal conclusion for economic growth policy.

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